Computational Macroeconomics

Module 3, 2017-2018

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Course description

Many, if not most, dynamic models used in modern macroeconomics do not have analytical (closed-form) solutions. For this reason, numerical methods and computer programming have become indispensable tools of the macroeconomic research. In this course we will discuss the main computational algorithms of the dynamic optimization problem. We will start with overview of the basic results from dynamic programming. Then, we will study the main numerical algorithms for its solution with application to a simple neoclassical growth model. In particular, we will consider linear-quadratic approximation, value and policy functions iterations algorithms and their modifications, perturbation and projection methods. Finally, we will discuss more complicated algorithms for solving heterogeneous agents' models, optimal policy and dynamic contracting problems. This course requires a basic knowledge of MATLAB, however programming skills in Fortran 90 or Python will be a plus.

Course requirements, grading, and attendance policies

There will be a few (maximum 4) home assignments (50% of the grade) asking for writing a code in MATLAB (or in GNU Octave, Fortran 90, Python, C++, etc.) to solve a simple dynamic programming problem. The exam (50% of the grade) will contain questions on a published macroeconomic article handed out in advance. All these components (including all home assignments), as well as at least 70% attendance, are mandatory for getting a passing grade.

Course contents

1. **Review of dynamic programming:** mathematical preliminaries, contraction mapping theorem, Blackwell's sufficient conditions, theorem of the maximum, dynamic programming under uncertainty

2. **Discrete-state dynamic programming:** value function iteration algorithm and its improvements, policy function iteration, interpolations and splines

3. **Linear approximation methods:** linear-quadratic (LQ) approximation algorithm, first-order perturbation methods

4. **Higher-order perturbation methods**

5. **Projection methods:** finite elements method, spectral methods (Chebyshev polynomials)

6. **Parameterized expectations algorithm**

7. **Heterogeneous agents models and incomplete market economies:** computation of stationary equilibrium, transitional dynamics, aggregate uncertainty in heterogeneous agents models, Krussel-Smith algorithm

8. **Extensions:** solving non-optimal economies, Ramsey policy under and without commitments, limited commitment and incentive problems

Sample tasks for course evaluation

Problem 1: Stochastic Growth Model

Consider the stochastic version of the neoclassical growth model:

$$v(k,z) = \max_{k' \in [0, f(k,z)]} \{u(f(k,z) - k') + \beta E\{v(k',z')|z\}\} \tag{FE}$$

Assume that utility and production functions are given by $u(c) = c^\gamma/\gamma$ and $f(k,z) = \exp(z)k^\alpha + (1-\delta)k$ and productivity shock z follows AR(1) stochastic process: $z' = \rho z + \epsilon'$ where ϵ' is i.i.d. $N(0,\sigma^2)$. Let $\beta = 0.9$, $\gamma = -1$, $\alpha = 0.3$, $\delta = 0.1$, $\rho = 0.85$ and $\sigma = 0.05$. Let $M = 100$ and $\mathcal{K} = \{k_1, k_2, \ldots, k_M\}$ where $k_0 = 0.01\bar{k}$ and $k_M = 1.5\bar{k}$ and $\bar{k}$ is the deterministic steady state level of capital and the distance between two consecutive points in $\mathcal{K}$ is constant.

1. Use Tauchen's method (see Tauchen, G (1986) 'Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions', Economics Letters: 20) to construct a 3-state Markov chain over $\mathcal{Z} = \{z_1, z_2, z_3\}$ that approximates the AR(1) process for z. Write down the Bellman equation for the discretized version of the model. *(1 point)*

2. Write MATLAB code that solves numerically the stochastic growth model by iterating over the value function. Provide two algorithms: with and without linear interpolation of the value function. Plot the computed value and policy functions. *(4 points)*

3. Assume that the model period is a quarter. Simulate the economy for 50 years, that is for 200 periods. Plot one stochastic realization of this economy, $\{y_t, c_t, i_t, k_t\}_{t=0}^{119}$ starting from $z_0 = 0$ and $k_0 = \bar{k}$. Use Hodrick-Prescott (HP) filter with $\lambda = 1600$

to compute and plot the deviations of the logs of simulated data $\{y_t, c_t, i_t, k_t\}$ about their HP(1600) trends. *(2 points)*

4. Obtain 100 independent stochastic realizations of this economy and write MATLAB code to compute the statistics that describe the business cycle fluctuations of $\{y_t, c_t, i_t, k_t\}$ (see Hansen, G.B. (1985) 'Indivisible Labor and the Business Cycle', Journal of Monetary Economics: 16; and Kydland F. E. and Edward C. Prescott (1990) 'Business Cycles: Real Facts and a Monetary Myth', Federal Reserve Bank of Minneapolis Quarterly Review (Spring): 318). *(3 points)*

Problem 2: LQ Approximation problem

Consider stochastic growth model with the *divisible* labor described in Hansen (1985):

$$\max_{\{c_t,n_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) \tag{1}$$

subject to

$$\begin{aligned} c_t + i_t &\leq z_t k_t^{\theta} n_t^{1-\theta}, \\ k_{t+1} &= i_t + (1-\delta)k_t, \\ z_{t+1} &= \gamma z_t + \epsilon_{t+1}, \ln(\epsilon_t) \sim N(\mu, \sigma^2) \end{aligned}$$

where $c_t \geq 0$, $0 \leq n_t \leq 1$ and k_0 is given. Assume that utility function is given by $u(c_t, 1-n_t) = \log(c_t) + A\log(1-n_t)$. Let $\beta = 0.99$, $A = 2$, $\theta = 0.36$, $\delta = 0.025$ and $\gamma = 0.95$.
Note, that productivity shocks have log-normal distribution with mean $E(\epsilon_t) = m = 1-\gamma$ and variance $Var(\epsilon_t) = v = 0.00712^2$. So, $\ln(\epsilon_t) \sim N(\mu, \sigma^2)$ where $\sigma^2 = \ln(\frac{v}{m^2} + 1)$ and $\mu = \ln(m) - \frac{1}{2}\sigma^2$.

1. Write MATLAB program that uses LQ approximation algorithm to replicate statistics reported for the *divisible* labor economy (third and fourth columns of Table 1 in the paper). *(5 points)*

2. Modify your program to replicate the statistics reported for the *indivisible* labor economy (fifth and sixth columns of Table 1 in the paper). Note, that representative agent in this version of the model has utility function given by $u(c_t, n_t) = \log(c_t) + B(1-n_t)$, where $B = -A\frac{\log(1-h_0)}{h_0}$ and $h_0 = 0.53$. *(5 points)*

Course materials

Required textbooks and materials

1. Heer, Burkhard & Maussner, Alfred, *Dynamic General Equilibrium Modeling: Computational Methods and Applications*, Springer, 2nd ed., 2009
2. Ljungqvist, Lars & Sargent, Thomas J., *Recursive Macroeconomic Theory*, The MIT Press, 2nd ed., 2004

Additional materials

1. Marimon, Ramon & Scott, Andrew, *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, 1999
2. Adda, Jerome & Cooper, Russell W., *Dynamic Economics: Quantitative Methods and Applications*, The MIT Press, 2003
3. Judd, Kenneth L., *Numerical Methods in Economics*, The MIT Press, 1998
4. Stokey, Nancy L., Lucas, Robert E. & Prescott, Edward C., *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989
5. DeJong, David N. & Dave, Chetan, *Structural Macroeconometrics*, Princeton University Press, 2nd ed., 2011

I will also provide a reading list of papers applying the quantitative methods discussed in the class, with the rate of about 2-3 per week.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.